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EXPLORING GRADE 11 LEARNERS' MATHEMATICAL CONNECTIONS WHEN SOLVING TRIGONOMETRIC EQUATIONS

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ABSTRACT

In this paper, we explored the intra-mathematical connections that grade 11 learners make when solving trigonometric equations. The study was guided by Mowat's theory of mathematical connections in which nodes and links are used to connect mathematical concepts and topics. We used a qualitative case study design within an interpretive paradigm to explore the intra-mathematical connections learners make as they solved trigonometric equations. The study was conducted in a high school in Mankweng Circuit, Limpopo Province, South Africa. Convenience sampling was used to select 30 learners who participated in the study. Data was collected using documents and task-based interviews. Data were analysed using inductive thematic analysis. The findings showed that learners made were able to make algebraic connections when solving trigonometric equations. They, however, were unable to make connections within trigonometry itself. This study, therefore, recommends that teachers stress the importance of connections when teaching trigonometry so that learners will not learn trigonometric concepts in isolation. In addition, it is recommended that further research be conducted on teaching strategies to improve learners' mathematical connection skills when solving trigonometric equations.

Keywords: Trigonometric equations, Links, Nodes, algebraic connections, trigonometric connections

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INTRODUCTION

Trigonometry is one of the important topics in the Further Education and Training (FET) phase in the South African mathematics curriculum. It provides basic knowledge and concepts that are used in other topics such as geometry, algebra and graphical ways of thinking (Sarac & Tutak, 2017). It is considered to improve learners' cognitive skills (Maphutha et al., 2022). Learning trigonometry prepares learners for advanced and challenging topics such as complex numbers, limits, derivatives, and integrals (Nabie et al, 2018). However, it is perceived to be one of the challenging topics in the South African FET phase (Ngcobo et al, 2019). The challenges in trigonometry could be explained by the multiple interrelated nature of the topic within itself and with other mathematics topics. As such learners find it difficult to link trigonometry with mathematical topics for example, geometry and algebra and real-life situations when solving trigonometric problems (Mosese & Ogbonnaya, 2021). Therefore, this requires that teachers guide learners in making mathematical connections during the teaching and learning process of trigonometry. The Curriculum and Assessment Policy Statement (CAPS) dictates that grade 11 learners learn the following trigonometric concepts: trigonometric identities, the reduction formulae, solutions of trigonometric equations, the sine, cosine, and area rules and solving problems in two-dimensions (Department of Basic Education (DBE), 2011). In this study, we focus on solutions of Trigonometric equations.

Solving Trigonometric equations requires learners to connect multiple interrelated mathematical concepts, algebraic processes and a variety of representation (Adhikari & Subedi, 2021). In addition, solving trigonometric equations requires learners to make connections between mathematical ideas, facts, procedures, and relationships within Trigonometry and across other mathematics topics (Hafiz et al, 2016). Owing to the interconnectedness nature of trigonometric concepts, learners experience challenges when solving trigonometry equations (Rohimah & Prabawanto, 2019). A number of researchers posted multiple observations about learners' struggles with solving trigonometric equations. For example, Rohimah and Prabawanto (2019) observed that learners have difficulties in deciphering the form of the problem, factorising and applying basic and quadratic trigonometric equations. The National Diagnostic Reports on South African examinations reveal that learners lack basic knowledge and skills for solving trigonometric equations (DBE, 2020 & 2021). According to Ernaningsih and Wicasari (2017), these challenges emanate from learners' inability to connect trigonometric concepts and use knowledge from other mathematical topics. Therefore, this calls for scrutiny of mathematical connection skills learners make when solving trigonometric equations in order to get in-depth knowledge about the problem.

In this study, we, therefore, envisaged exploring Grade eleven learners' mathematical connections when solving trigonometric equations. Making mathematical connections is a fundamental skill (Kenedi et al., 2019; García-García & Dolores-Flores, 2020) or tool (Pambudi et al, 2020) needed at each stage of problem-solving in Mathematics (Kleden et al, 2021). It is a cognitive process that involves establishing links between mathematical ideas. The ability to make mathematical connections is an integral component of successful problem-solving (Cichon & Ellis, 2020). Learning a mathematical concept and considering how it originates, extends, and connects with other concepts across the grades helps learners to develop a deeper understanding. However, Nurmeidina and Rafidiyah (2019) observed that learners lack skills of connecting mathematical concepts during problem-solving. An emphasis on mathematical connections helps learners to recognise how ideas in different topics and subjects are related. The focus of this study was to explore the mathematical connections grade 11 learners make when solving trigonometric equations.

Mathematical connections are variously defined in the literature. Siregar and Surya (2017) defined mathematical connections as relationships between mathematical ideas. According to Rahmi and Subianto (2020), mathematical connections are described as a network of concepts forming a spider's web-like structure which consist of nodes, which represent main concepts and thread-like structures called links which represent connections or relationships between them. Making mathematical connections is a constructivist process in which learners organise mathematical ideas into a coherent system or schemata (Suominen, 2018). The formation of a schema for a mathematics concept depends on the ability to make connections (Yang et al., 2021). Kenedi et al (2019) define mathematical connections as the ability to recognize and use connections among mathematical ideas; understand how mathematical ideas interconnect and build on one another to produce a coherent whole; recognize and apply mathematics in contexts outside of mathematics. In the context of this study, mathematical connections are understood as associations, or relationships between concepts, representations, processes and relevant links that learners make during problem-solving (Richland et al., 2012).

Mathematical connections require learners to look at their solutions and reflect (Evans & Swan, 2014). What a learner notices in her or his solution links to current or prior learning, helps that learner to discover new learning and relates the solution mathematically to his/ her own world. A learner is considered mathematically proficient when s/he is able to make mathematical connections linking both the mathematics and the situation in the task. According to Siregar and Siagian (2019), mathematical connections are viewed as broad ideas/processes that can be used to link different topics in mathematics. A mathematical connection can be conceptualized as an artefact of the learning process itself. In other words, making a mathematical connection is a process that occurs in the mind of the learner(s) and the connection is something that exists in the mind of the learner; it is a mental construction of the learner.

There are three broad categories of mathematical connections. Mathematical connections that learners can make between mathematics and everyday life are referred to as extra-mathematical connections (Quiros, 2016). Whereas connections between mathematical concepts and other subjects are referred to as inter-mathematical connections and the relations within mathematics, understanding how the mathematical concepts link and build on each other to make a unified whole are called intra-mathematical connections (Putri & Wutsqa, 2019). In this study, we explored the intra-mathematical connections that learners make when solving trigonometric equations because the focus was on the links within mathematical concepts. This study explored the intra-mathematical connections learners make when solving trigonometric equations problems. Learners are expected to connect concepts within mathematics and trigonometry in order to solve trigonometric equations successfully (Yosopranata et al., 2018). Without making relevant links or connections among concepts within mathematics, learners will be ill-equipped to effectively solve mathematical problems (Rodríguez-Nieto et al., 2022). Wide knowledge of various mathematical concepts is significant for making intra-mathematical connections during problem-solving (Usman & Hussaini, 2017). In addition, the ability to make intra-mathematical connections plays an important role in building an understanding of mathematical concepts (Zengin, 2019). Accordingly, intra-mathematical connections help learners to link concepts and ideas in mathematics which allow lifelong and deep understanding and view of mathematics as a coherent whole (García-García & Dolores-Flores, 2020).

To solve trigonometric equations correctly, learners must have skills in making intra-mathematical connections, following correct procedures, and using multiple representations (Martín-Fernández et al., 2019). However, Hafiz et al. (2016) indicated that learners have difficulties in making intra-mathematical connections when solving trigonometric equations. Jailani et al. (2020) reported that learners' difficulties in making intra-mathematical connections are caused by a lack of depth in mathematics. According to Chigonga (2016), Trigonometric equations are still perceived as complex to solve, and learners do not understand some of the mathematical concepts such as the conditions for which some solutions exist, which are normally expressed as inequalities. Furthermore, he states that learners fail to draw diagrams representing the equation in suitable quadrants. In addition, some learners do not consider or understand that solutions in trigonometric equations are periodic and infinite (Mutodi, 2016).

Yumiati and Haji (2018) noted that studies on intra-mathematical connections of specific topics are still lacking. Most studies in this field explored mathematical connections that in-/pre-service teachers make when teaching mathematics (Garcia-Garcia & Dolores-Flores, 2019; Nieto, Rodríguez & García, 2021). However, studies on learners, focused on their efficacy in making connections (Trihatun, 2019; Jingga et al, 2019), instead of the mathematical connections that they are making. Garcia-Garcia and Dolores-Flores (2020) suggested that it is significant to explore the mathematical connections learners make in order to establish the extent to which they are able to dig into their knowledge base during problem-solving. Such depth in knowledge provides a rich source of ideas that are useful in solving mathematics problems.

In this study, we envisaged answering the following research question: What mathematical connections do grade 11 learners make when solving trigonometric equations? We argue that identifying learners' mathematical connections when solving trigonometric problems will help in establishing how they link ideas to arrive at solutions. Making mathematical connections is a skill that must be inculcated in learners during the teaching and learning of trigonometry. In so doing, learners will be able to establish that trigonometry is an integrated and interconnected topic. This will alleviate the problem of poor performance in problematic trigonometry concepts which learners treat as isolated. The findings of this study will help teachers to realize the mathematical connection skills that learners make when solving trigonometric problems. Teachers, therefore, will be able to support learners in making those connections.

Theoretical Framework

This study is guided by the Network theory (Mowat and Davis, 2010). The Network theory consists of a connected web representing relationships of concepts or ideas for solving or understanding a mathematical concept. The theory postulates that mathematical ideas can be integrated through complex linkages between concepts (Mowat and Davis, 2010). Mowat and Davis (2010) defined a mathematical concept as a complex unit that emerges from the interaction of elements called concepts (nodes) that are connected by conceptual metaphors called links. Mathematical concepts can be perceived as a system of nodes (main concepts) in a network and their interactions as links among nodes (Barabási, 2003). Thus, the theory views problem-solving as a system that has nodes in a network and their interactions as links.

The theory considers mathematics as a network or web of ideas, concepts and processes that are connected (Doll, 2010). According to Mowat and Davis (2010), the theory views main mathematical concepts as nodes in a network connected by links. Mowat (2008) defines nodes as networks themselves and represents a set of insights and relationships involving the concept. Links in the network of mathematics are conceptual relations that connect from one domain (the source) and are mapped onto corresponding aspects of another domain (the target). Further, the network theory examines ways in which nodes of knowledge in mathematics as a discipline can be linked (Mowat & Davis, 2010). In the context this of study, a network refers to how learners organise ideas/ concepts as they solve problems. The theory emphasised that concepts do not develop in isolation but through the connection to other concepts and ideas (Mowat, 2008). As a result, the network structure explains how the concepts in Mathematics and within trigonometry to are connected trigonometric equations. Figure 1 depicts the network structure of nodes and links for solving trigonometric equations.

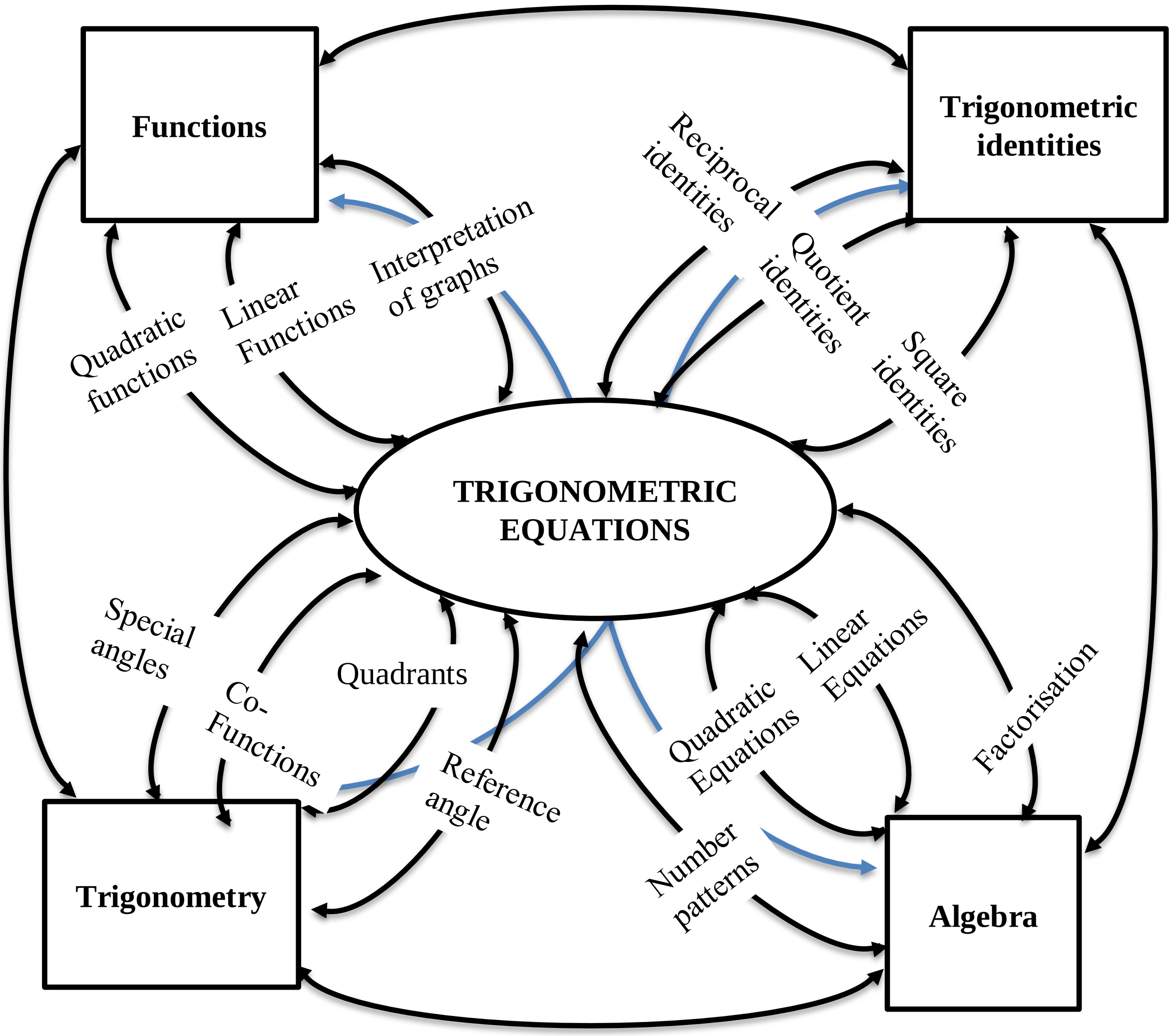


Figure 1: A network of concept for solving trigonometric equations.

The central concept or main node in this study is solving trigonometric equations and the sub-nodes consist of trigonometry, functions, trigonometric Identities, and algebra.

Concepts and processes such as factorisation, quadratic equations, linear equations, inequalities, generalization and number patterns link the Algebra and Trigonometric equations nodes. Learners who are able to make links between Algebra and Trigonometric equations nodes can convert trigonometric equations into algebraic quadratic or linear forms which are simpler to solve using algebraic processes such as factorization, using the quadratic formula, pattern recognition and generalization.

The functions and trigonometric equations nodes are connected are linked by concepts such as functions, interpretation of functions, range, and domain, sketching graphs, period and inequalities. A learner who possesses graphical skills is able to connect knowledge of functions when solving trigonometric equations (Ngu & Phan, 2020). The strategy is to obtain an initial solution and then work with the graph and its symmetries to find additional solutions.

Knowledge of trigonometric identities is another node that is essential for solving trigonometric equations. This node is connected to trigonometric equations by links such as compound angle identities, double angle identities, square identities, reciprocals and quotient identities. The knowledge of fundamental identities makes solving trigonometric equations simpler (Kamber & Takaci, 2018). These equations often require the use of trigonometric identities to solve for the unknown variable. Since the trigonometric functions are periodic, there are infinitely more solutions, which need learners to apply their knowledge of generalisations.

Knowledge of concepts within trigonometry is an essential tool for solving trigonometric equations. The links connecting these nodes are concepts such as reference angles, quadrants, co-functions and special angles. To solve trigonometric equations effectively, learners should have a good grasp of the trigonometric ratio values in the first quadrant, how the unit circle works, the relationship between radians and degrees, and what the various trigonometric functions graphs look like, at least on the first period. Solving trigonometric equations uses both reference angles and general solutions. To find reference angles, learners need knowledge of inverses of trigonometric functions and the CAST diagram while general solutions require learners to possess knowledge of generalisation as well as linear number patterns. Lastly, it is important to note that the nodes and links are not linearly related, that is, ideas are structured in a web-like structure where they complement each other.

METHODS

In this paper, we used a qualitative approach within an interpretive paradigm (Creswell & Poth, 2016). An interpretive case study was used to describe and interpret the data collected richly and thickly (Merriam, 1998). Convenience sampling was used to select 30 Grade 11 learners from a non-fee-paying school in Mankweng Circuit, Limpopo province in South Africa to participate in this study due to their easy accessibility. The sample consisted of 15 male and 15 female learners with ages ranging from 16 – 17 years old. Grade 11 class was found suitable as trigonometric equations are prescribed to them. Therefore, this sample was found appropriate as it could provide enough data to get the conclusions.

Data were collected through written tasks in their classwork books as documents and task-based interviews. These documents allowed us to identify the intra-mathematical connections that learners make in their written responses to the tasks given during the lessons. The task-based interviews were conducted during the lessons with each individual learner who portrayed mathematical connections when solving the given activity. These types of interviews were unstructured as they depended on what the researcher saw fit to ask depending on the learner's solution or process of finding the solution. The task-based interviews allowed for asking detailed questions and probing participants to provide rich data related to the task responses. The learner and researcher interacted on a task. As the learner solved the equation in writing the researcher looked into their justifications and posed questions to identify the mathematical connections they make when solving the equations. Data were analysed using inductive thematic analysis (Braun, & Clarke, 2006). Data were re-read made so as to make sense of the emerging ideas. We coded the ideas and arranged them to form the patterns. Categories were developed from the patterns. The themes which are reported as final results in this paper emerged from the categories identified. Ethical clearance was sought with the Limpopo Province Department of Education. Learners' parents gave their consent for their children to participate in the study. Learners assented themselves to participate in the study.

RESULTS AND DISCUSSIONS

The following themes and sub-themes emerged from the data:

Table 1. Themes and sub-themes that emerged from the data

Themes	Sub-themes
Learners' trigonometric connections between concepts	Inability to apply the co-function concept
	Inappropriate application of the square identity
	Inadequate knowledge of quadrants and interval concepts
Learners' algebraic connections between concepts	Ability to solve linear trigonometric equations.
	Ability to solve quadratic trigonometric equations

Theme 1: Learners' trigonometric connections between concepts
Sub-theme: Inability to apply the co-function concept.

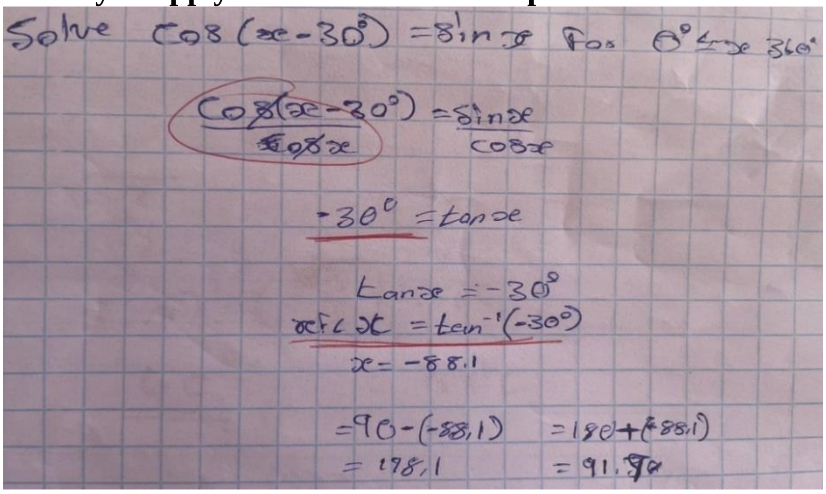


Figure 1.2: Learner's inability to apply the co-function concept.

The activity item $\cos(x - 30°) = sinx$ required learners to use the co-function of one of the trigonometric ratios given. The learners were able to notice that they should have a single trigonometric ratio in order for them to be able to solve the given problem. Some learners decided to get rid of two different trigonometric ratios. For example, the learner in figure 1.2 divided both sides by the $\cos x$ and thereafter converted the $\frac{sin\,x}{cosx}$ to $tanx$ in order to have a single trigonometric ratio. This shows that the learner used the wrong link of the quotient identity when trying to link the trigonometric identity node with the trigonometric equations node. After dividing both sides by $\cos x$, the learner in figure 1.2 cancelled $\cos x$ in the numerator with the one in the denominator. For her, $\cos(x - 30°)$ is not one term. She failed to link algebra and trigonometry nodes. When asked why she divided by $\cos x$ on both sides, the learner responded by saying "*I want to have a single trigonometric ratio which is the tangent because the $\cos x$ in the numerator will cancel $\cos x$ in the denominator in the right-hand side*". This learner's response revealed the learner's inability to link the co-function concept to the equation given. These results are consistent with Nurmeidina and Rafidiyah's (2019) observations that one of the difficulties learners encounters is the correct use of concepts when solving mathematical problems. In addition, the results are similar to Ernaningsih and Wicasara (2017) who found that learners' difficulties to solve trigonometric problems emanate from their inability to make connections within trigonometry itself and between mathematics topics.

Sub-theme: Inappropriate application of the square identity

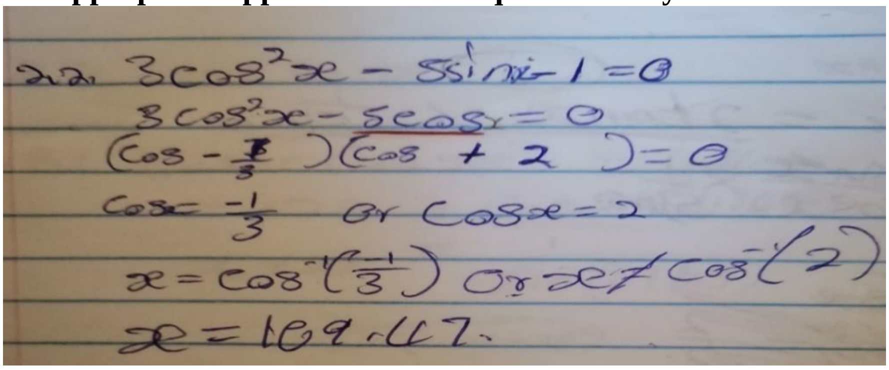


Figure 1.3: Learner's inability to apply the square identity to solve the trigonometric equation.

Some learners were able to recognise that the given problem will lead to a quadratic equation. They realised that they should convert one ratio to the other so as to satisfy the condition of the quadratic equation. Instead of converting cos^2x to $1 - sin^2x$, the learner in figure 1.3, substituted $sinx - 1$ by $cosx$. This learner was trying to link the square identity to the equation. He, however, disregarded the rule of factors of the difference of two squares. He thought that $1 - sin^2x = sin^2x - 1$. During interviews, this learner said, *"What I know is that the square root of x squared is x, therefore, the square root of* $sin^2x - 1 = sinx - 1$. *I then substituted* $sinx - 1$ *by* $cosx$ *because* $sin^2x + cos^2x = 1$*."* This reveals the learner's inability to link Algebra to trigonometric equations. These findings concur with Rohimah and Prabawanto (2019) who noted that learners have difficulties in factorising quadratic trigonometric equations and doing algebraic manipulations due to lack of prior knowledge of algebraic procedures. These results are consistent with Jailani et al. (2020) who observed that the lack of depth of knowledge of mathematical concepts hinders learners to make mathematical connections when solving problems.

Sub-theme: Inadequate knowledge of quadrants and interval concepts in trigonometry

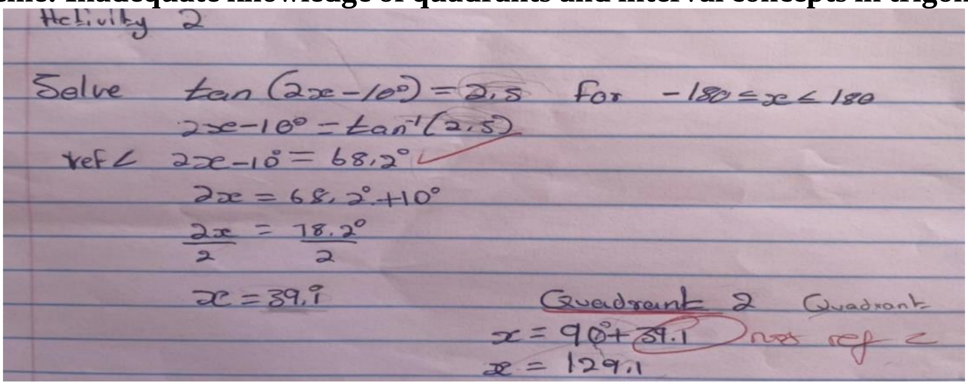


Figure 1.4: Learner's lack of quadrants and interval knowledge

Most learners were able to link trigonometric equations to trigonometry itself. They however were unable to link the calculated reference angle to the quadrants where the solution will be found. The learner in Figure 1.4 solved the equation by linking the trigonometry and trigonometric equation nodes using the inverse and reference angle concepts. However, the learner did not consider the given interval, hence he got only one correct solution to the problem. When asked why he added $39{,}1°$ to $90°$, the learner responded by saying *"What I know is that the period of the tangent is* $90°$ *so as the reference angle is* $39{,}1°$*, then another angle will be* $90° + 39{,}1°$*"*. This response revealed that the learner is unable to link the reference angle to the quadrants and interval given. These results are consistent with Ernaningsih and Wicasara (2017) who found that learners' difficulties to solve trigonometric problems emanate from their inability to make connections within trigonometry itself and between mathematics topics. In addition, the results are similar to Nurmeidina and Rafidiyah's (2019) observations that one of the difficulties learners encounters is the correct use of concepts when solving mathematical problems.

Theme 2: Learners' algebraic connections between concepts

Sub-theme: Learners' ability to solve linear trigonometric equations

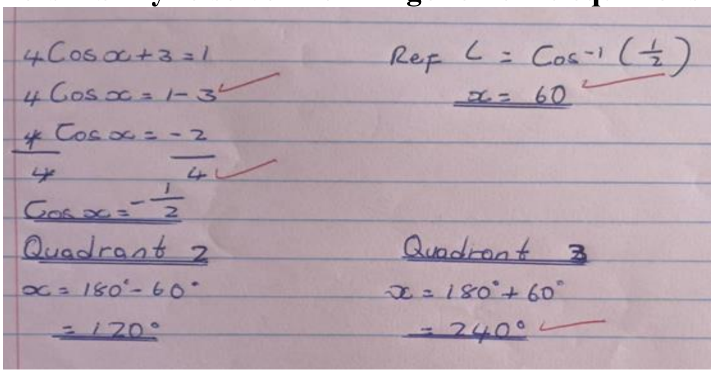
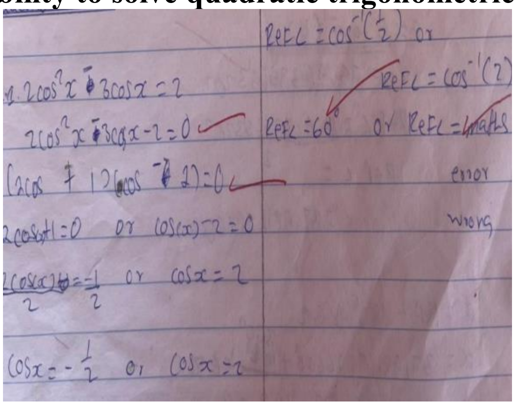


Figure 1.5: Learner's ability to solve linear trigonometric equation $4cosx + 3 = 1$

When solving the equation $4cosx + 3 = 1$, most learners linked it with algebra. Most learners used the link of linear functions from the algebra node to solve the trigonometric equation. Most learners were able to work out the problem algebraically until getting the correct reference angle and the solutions of the equation. During the task-based interviews, when asked how he managed to get the solutions, the learner said "*I treated the equation like it is a linear equation of* $4x + 3 = 1$, *where* $cosx$ *is represented by x in the equation. I then added the additive inverse of 3 both sides and then divided both sides by the coefficient 4 of* $cosx$ *in order to remain with* $cosx$ *in the right-hand side. Then I found the reference angle which led me to the solutions.*"

These results are consistent with Sarkam et al. (2019) who found out that learners were able to make links between different mathematical concepts in solving trigonometry problems. Learners were able to use the trigonometry formula and appropriate algebraic operations. The results are in line with Hafiz et al. (2016) who affirm that learners find difficulty in connecting the trigonometric problems with the previously learnt concepts. They have difficulty in solving trigonometric problems regardless of having learnt similar problems in algebra previously.

Sub-theme: Learners' ability to solve quadratic trigonometric equations



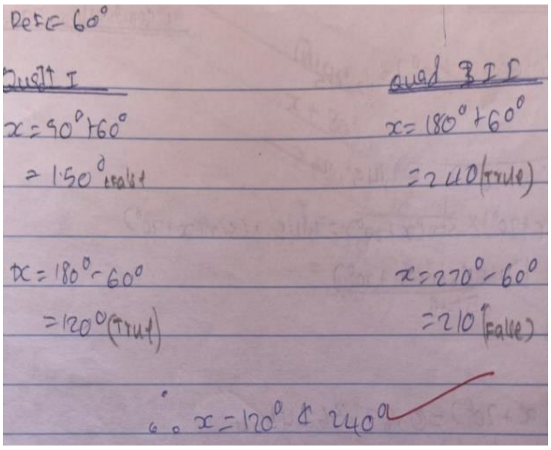


Figure 1.6: Learner's ability to solve quadratic trigonometric equation
$2cos^2x - 3\cos x = 2$

It is evident from most learners' responses of the equation $2cos^2x - 3\cos x = 2$ that they had the ability to make link the nodes of algebra and trigonometric equations. The learners linked the given equation with quadratic equations. The learners added the additive inverse of positive 2 on both sides. They were also able to algebraically find the correct factors of the equation. At first the learner did not get the factors correct. However, when asked about the factors during the task-based interviews the learner responded by saying *"No, I think I have made a mistake on the signs here ...pointing inside the brackets. I concentrated on the value of c and forgot to check the value of b."* This learner was referring to b and c as the coefficients of x^2 and x in the general quadratic equation $ax^2 + bx + c = 0$.

These results are consistent with Sarkam et al. (2019) who found that learners were able to make links between different mathematical concepts when solving trigonometry problems. These findings contradict Rohimah and Prabawanto (2019) who noted that learners have difficulties in factorising quadratic trigonometric equations and doing algebraic manipulations due to a lack of prior knowledge of algebraic procedures. The results are also contrary to Ngu and Phan (2020) who affirm that learners are not able to link algebraic concepts to solve trigonometric problems.

CONCLUSIONS

The findings from learners' classwork documents and task-based interviews revealed the ability of learners to make links between the learnt node of algebra and trigonometry which helped them to find the solutions. The links observed were linear and quadratic equations which served as learners' prior knowledge for solving the given trigonometric equations. The ability to make algebraic connections enabled learners to make appropriate procedures such as factorising the quadratic trigonometric equations until they got the correct reference angle which led them to correct solution. The findings also highlighted that learners were unable to link the given equation within the trigonometry node. This indicates that these learners are not aware that trigonometry is integrated within itself. When solving the given trigonometric equations, learners were unable to apply the co-function concept, showed inadequate knowledge of quadrants and interval concepts and applied the square identity inappropriately. These affected learners' solutions to the given equations.

RECOMMENDATION

The findings of this study indicate that learning through making mathematical connections needs to be strengthened to improve mathematics learning and teaching. The findings of this study provided in-depth knowledge about the mathematical connections which learners make when solving trigonometric equations. This could inform strategies to improve learners' mathematical connection skills to solve trigonometric equations problems effectively. It is recommended that mathematical connections between concepts be encouraged, and learners should be guided in making such connections. In addition, we recommend that teachers stress the importance of the integration of concepts when teaching trigonometry so that learners will not learn trigonometric concepts in isolation.

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