Project-based learning in analytical geometry: Problem-solving and representational reasoning

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Abstract

Developing meaningful problem-solving approaches remains a central challenge in undergraduate Analytical Geometry courses. This study explores students' learning experiences and approaches to problem-solving following the implementation of Project-Based Learning (PjBL) in an Analytical Geometry course. A mixed-methods design with a convergent parallel approach was adopted to provide a comprehensive understanding of the learning process. Research data were obtained from a questionnaire focusing on students' experiences during project-based activities and learning outcome scores. The participants consisted of 22 students from a Mathematics Education program during one semester of instruction. The research results showed high student responses toward the lecturer's presentation, the usefulness of instructional tasks/media, and lecture satisfaction. In contrast, lower scores were observed for the lecturer's interpersonal approach/friendliness and the interestingness of the material course content. Students reported using efficient strategies supported by visualisation tools, such as GeoGebra, to better understand problem structures. The integration of findings suggests that PjBL provides a learning environment that supports reflective engagement with problem-solving processes in analytical geometry.

Keywords: analytical geometry, mathematics learning, mathematical problem-solving, project-based learning

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INTRODUCTION

Analytical Geometry plays a central role in undergraduate mathematics education because it requires students to coordinate algebraic manipulation, geometric visualization, and deductive reasoning within a single problem-solving process. Unlike introductory algebra or calculus, analytical geometry demands the flexible use of multiple representations to interpret and solve problems meaningfully. Consequently, students' success in this course is closely related to their ability to engage in non-routine problem-solving rather than the application of isolated procedures.

Despite its importance, prior research has consistently reported that students experience significant difficulties in learning analytical geometry, particularly when instruction emphasizes procedural

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fluency over conceptual understanding and strategic reasoning. Studies grounded in the van Hiele theory of geometric thinking suggest that many students remain at lower levels of geometric reasoning when learning environments do not explicitly support visualization and conceptual connections (van Hiele, 1986). Similarly, Boaler (2002) argues that procedural teaching practices tend to limit students' opportunities to develop flexible problem-solving approaches, leading to an overreliance on memorization and imitation.

From a broader perspective, problem-solving is widely recognized as a core goal of mathematics education. The National Council of Teachers of Mathematics (NCTM, 2000) positions problem-solving not merely as a topic, but as a fundamental process through which mathematical understanding is constructed. However, research in higher education contexts indicates that students often struggle to transfer procedural knowledge to novel problem situations, particularly in mathematically abstract domains such as analytical geometry. This gap between procedural competence and strategic problem-solving highlights the need for instructional approaches that promote active engagement, exploration, and reflection.

Project-Based Learning (PjBL) has been widely advocated as a pedagogical approach capable of fostering higher-order thinking skills, including problem-solving, critical thinking, and creativity. Within PjBL environments, students are encouraged to work on extended tasks that require inquiry, collaboration, and the application of mathematical ideas to meaningful contexts (Thomas & Mergendoller, 2000; Serin, 2023; Gao & Zhang, 2023; Tyata et al., 2021; Lee, 2022; Guizado, Ortiz & Abanto, 2024). Empirical studies in mathematics education have shown that PjBL can enhance students' motivation and engagement, while also supporting the development of conceptual understanding and problem-solving abilities (Holmes & Hwang, 2016; Rehman et al., 2023; Fülöp, 2021; Ozkan, & Topsakal, 2020; Mahanal et al., 2022).

In the context of geometry learning, PjBL offers particular advantages because project activities often involve visual exploration, model construction, and the interpretation of multiple representations. The integration of dynamic mathematics software, such as GeoGebra, further strengthens this approach by enabling students to manipulate geometric objects and observe invariant relationships in real time. Prior studies have demonstrated that technology-supported PjBL can help students connect algebraic expressions with geometric representations and improve their understanding of geometric concepts (Surya et al., 2021; Kholid et al., 2022).

However, existing research on PjBL in mathematics education has predominantly focused on learning outcomes, motivation, or general problem-solving performance, often at the school level. Relatively fewer studies have examined undergraduate-level analytical geometry courses, where abstraction and representational complexity are substantially higher. Moreover, many studies rely on quantitative measures alone, providing limited insight into how students' problem-solving approaches develop during project-based activities.

Although prior research provides evidence that Project-Based Learning can support students' mathematical learning, several important gaps remain. First, there is limited empirical research that examines how PjBL shapes students' problem-solving approaches in analytically demanding courses such as analytical geometry at the undergraduate level. Most studies report improvements in test scores or general achievement without explicating the cognitive and strategic processes underlying these outcomes.

Second, research on mathematical problem-solving has emphasized the distinction between imitative reasoning and creative or strategic reasoning (Lithner, 2008). However, few studies have explored how instructional approaches like PjBL facilitate students' transition from procedural imitation toward more flexible and efficient problem-solving strategies, particularly through the coordination of algebraic and visual representations.

Third, despite growing interest in mixed-methods research in mathematics education, the integration of quantitative and qualitative findings remains underutilized. Many studies collect multiple data sources but stop short of systematically integrating them to explain learning processes. As a result, the mechanisms through which PjBL influences students' problem-solving experiences often remain implicit rather than analytically articulated.

Prior research has emphasized the importance of instructional designs and learning materials that actively engage students in constructing mathematical understanding. Studies on the development of student worksheets and activity-based learning resources have shown that well-designed tasks can support students' mathematical connections, communication, and strategic thinking, particularly in geometry-related topics (Kosasih, Nurjanah, Saputra, & Mutmainnah, 2023; Ulpah, Kosasih, & Saputra, 2023). These findings suggest that learning environments which encourage exploration, representation, and reflection provide a foundation for the development of meaningful problem-solving approaches. In a similar vein, recent studies published in the Uninus Journal of Mathematics Education and Science report that Project-Based Learning can enhance students' mathematical problem-solving abilities through contextual and collaborative activities, even at earlier educational levels (Prafita et al., 2025; Difinubun, Makmuri, & Hidayat, 2024; Halim, 2025; Chhabra & Gawande, 2025; Prayekti, 2025). Nevertheless, empirical evidence remains limited regarding how such approaches operate in analytically demanding undergraduate courses, particularly in analytical geometry, where representational complexity and abstraction pose additional challenges.

Based on this background, this study aims to answer the following research questions: 1) What is the impact of implementing Project-Based Learning in Analytical Geometry on students' problem-solving skills?, 2) What are the characteristics of students' problem-solving strategies that develop during the implementation of Project-Based Learning?, 3) How do quantitative and qualitative findings complement each other in explaining the mechanisms of project-based learning in the context of Analytical Geometry?

In response to these gaps, this study aims to examine the implementation of Project-Based Learning in an undergraduate Analytical Geometry course, with particular attention to students' approaches to mathematical problem-solving. Using a mixed-methods design with a convergent parallel approach, this study integrates quantitative data on student perceptions and learning outcomes with qualitative data on students' learning experiences and problem-solving strategies ((Prafita et al., 2025; Difinubun, Makmuri, & Hidayat, 2024; Halim, 2025; Kosasih, & Saputra, 2023).

This study makes several contributions to the literature. Empirically, it provides evidence from a higher education context, specifically analytical geometry, which remains underrepresented in PjBL research. Methodologically, it demonstrates the use of joint displays to integrate quantitative and qualitative findings, thereby offering a more comprehensive account of students' problem-solving processes. Theoretically, the study contributes to discussions on mathematical problem-solving by illustrating how project-based activities and dynamic visualization tools can support the emergence of flexible and efficient problem-solving strategies grounded in conceptual understanding. By addressing both learning outcomes and learning processes, this study seeks to enrich current understandings of how Project-Based Learning can be designed and implemented to support meaningful problem-solving in undergraduate mathematics education.

This study focused on mathematics education students taking Analytical Geometry at a single higher education institution. The scope of the study was limited to the development of mathematical problem-solving skills in the context of project-based learning, without conducting inferential comparisons between groups or broad statistical generalizations. The findings of this study are intended to provide a contextual and mechanistic understanding of the implementation of Project-Based Learning, not to generate universal causal claims.

METHODS

This study employed a mixed-methods approach using a convergent parallel design. This design was selected to obtain a comprehensive understanding of the implementation of Project-Based Learning (PjBL) in an undergraduate Analytical Geometry course by integrating quantitative trends with qualitative insights into students' learning experiences. The convergent design allows quantitative and qualitative data to be collected during the same phase of the study, analyzed separately, and then integrated to generate meta-inferences (Creswell & Plano Clark, 2018). The rationale for using a mixed-methods approach lies in the complexity of mathematical problem-solving. Quantitative data alone are insufficient to capture students' strategic reasoning processes, while qualitative data without quantitative support may lack generalizability. Therefore, integrating both data types enables a more nuanced explanation of how PjBL supports students' approaches to problem-solving in analytical geometry.

The participants consisted of 22 undergraduate students enrolled in the Mathematics Education program at Universitas Islam Nusantara, Indonesia. All participants were registered in an Analytical Geometry course conducted over one academic semester. A total sampling technique was employed, as all students in the class participated in the study and experienced the same instructional design. The inclusion criteria included: (1) active students who attended the entire series of project-based lectures, (2) minimum attendance of 80% during lectures, and (3) completion of all project assignments and research instruments. Students who did not meet these criteria were not involved in the data analysis.

The Analytical Geometry course covered topics that are conceptually demanding and highly representational, including equations of lines and planes, distances and intersections in three-dimensional space, and coordinate representations. Throughout the course, PjBL was implemented consistently across multiple learning units. Dynamic mathematics software, particularly GeoGebra, was integrated into the projects to support visualization and exploration of geometric relationships.

Quantitative data consisted of two components:

1. Student Perception Questionnaire

A closed-ended questionnaire using a 7-point Likert scale was administered to measure students' perceptions of the PjBL implementation. The questionnaire included items related to instructional methods, lecturer delivery, learning media, and overall learning satisfaction. A 7-point scale was chosen to increase sensitivity to variation in student responses.

2. Learning Outcome Data

Learning outcome data were obtained from course assessments, including project assignments, mid-semester examinations, and final examinations. These data were used descriptively to provide contextual information about students' academic performance following the implementation of PjBL. Importantly, these scores were not treated as direct measures of problem-solving ability, but rather as indicators of overall course achievement.

Qualitative data were collected through an open-ended questionnaire administered at the end of the course. The questionnaire invited students to reflect on their learning experiences during PjBL activities, including perceived benefits, challenges encountered, and approaches used to solve analytical geometry problems.

This qualitative data source was selected to capture students' perspectives on how project-based activities influenced their engagement, representational use, and problem-solving strategies—dimensions that are difficult to observe through quantitative instruments alone. The closed-ended questionnaire was developed to capture students' perceptions of the PjBL learning environment. The items addressed four main aspects: clarity of instruction, attractiveness of learning methods, effectiveness of learning media, and learning satisfaction. The questionnaire instrument was adapted from the Project-Based Learning framework developed by PBLWorks. To ensure contextual relevance, the questionnaire items were reviewed through expert judgment involving two mathematics education lecturers experienced in project-based learning and mathematics pedagogy.

Both quantitative and qualitative data were collected at the end of the instructional period. The closed-ended and open-ended questionnaires were administered simultaneously to all participants after the completion of the course. Learning outcome data were obtained from institutional records following the conclusion of mid-semester and final examinations. Collecting data at the same time point ensured consistency in students' reflections and supported the convergent design of the study.

Operationally, the research procedure was carried out through the following stages: 1) Project-based learning planning and development of research instruments; 2) Implementation of Project-Based Learning in Analytical Geometry courses; 3) Quantitative data were analyzed descriptively to identify trends in students' responses and learning outcomes, while qualitative data were analyzed thematically to explore students' problem-solving experiences and representational reasoning.; 4) Qualitative data collection through student reflections and project documentation; 5) Parallel quantitative and qualitative data analysis; and 6) Integration of findings through joint displays.

Quantitative data were analyzed using descriptive statistical techniques, including means, percentages, and categorical distributions. Questionnaire responses were summarized to describe students' perceptions of the PjBL implementation. Learning outcome data were categorized into low, medium, and high achievement levels to provide an overview of students' academic performance. No inferential statistical analysis was conducted, as the study did not aim to test causal relationships or compare groups.

Qualitative data were analyzed using thematic analysis following the procedures outlined by Braun and Clarke (2006). The analysis involved several stages: initial familiarization with the data, generation of initial codes, identification of recurring patterns, and development of broader themes. Particular attention was given to students' descriptions of problem-solving strategies, the use of visualization tools, and reflections on project activities. These themes were interpreted in relation to theoretical perspectives on mathematical problem-solving and representational reasoning.

Integration occurred during the interpretation phase through the use of joint displays (Guetterman et al., 2015). Quantitative and qualitative findings were juxtaposed to examine convergence, divergence, and complementarity between data sources. This integration enabled the identification of patterns that could not be fully explained by a single type of data, thereby strengthening the explanatory power of the findings. Quantitative data analysis was conducted descriptively to describe trends and variations in scores, without conducting inferential statistical tests, in line with the exploratory objectives of the study and the limited sample size.

The credibility of the study was enhanced through methodological triangulation, as multiple data sources were used to examine the same phenomenon. The mixed-methods approach also allowed for cross-validation between quantitative trends and qualitative insights. Transparency in data collection and analysis procedures further supports the trustworthiness of the findings.

This study adhered to ethical principles in educational research. Participation was voluntary, all data were anonymized, and the results were used solely for academic purposes. No conflicts of interest were identified. This study has methodological limitations, including a limited sample size and a single institutional context, so the findings are not intended to be statistically generalizable. However, the mixed-methods design allows for an in-depth exploration of the processes and mechanisms of project-based learning in the context of Analytical Geometry.

RESULTS AND DISCUSSION

Implementation of Project-Based Learning

The implementation of Project-Based Learning (PjBL) in this study was adapted from the framework developed by PBLWorks (n.d). The learning process was conducted over one semester in an undergraduate Analytical Geometry course and emphasized student-centered investigation, collaborative problem-solving, and project development. The implementation followed several main stages: (1) introduction of essential geometric problems, (2) project planning and group discussion, (3) exploration and development of mathematical representations, (4) project presentation and peer feedback, and (5) reflection and evaluation.

During the learning process, the lecturer acted as a facilitator by introducing contextual geometric problems, guiding students' investigations, supporting mathematical discussions, and providing feedback throughout project development. The lecturer also encouraged students to compare alternative solution strategies and to use multiple representations in explaining geometric concepts. Students worked collaboratively in small groups to analyze geometric problems, construct visual and symbolic representations, develop project outputs, and communicate their reasoning through presentations and reflective discussions. The project activities encouraged students to explore different approaches to problem solving and to connect geometric concepts with real mathematical situations.

Results of the Closed Questionnaire on Student Perceptions of PjBL

A closed-ended questionnaire with a Likert scale of 1–7 consisted of six statements, completed by 22 students. The resulting data were analyzed descriptively, calculating the total score, dominant score, average score, and interpretation.

Table 1. Summary of Closed Questionnaire Results for Students on PjBL Learning

No	Statement	Total Score	Number of Respondents	Average	Standard Deviation	Dominant Score	Interpretation
1	Satisfaction with lectures	132	22	6.00	0.67	6–7	Students were satisfied with the learning process.
2	The general interest of the lecture	132	22	6.00	0.93	6–7	The majority of students considered the lectures interesting.

No	Statement	Total Score	Number of Respondents	Average	Standard Deviation	Dominant Score	Interpretation
3	The interest of the lecturer's presentation	137	22	6.23	0.75	6–7	The lecturers' presentation methods were considered very engaging.
4	The friendliness of the lecturer in presenting the lecture	76	22	3.45	2.86	1	The lecturers' friendliness was deemed to still need improvement.
5	The interest of the material presented	74	22	3.36	2.32	1	The material was not entirely engaging for students.
6	The media (tasks) used	133	22	6.05	1.00	6–7	The project media was deemed to be very supportive of learning.

Based on Table 1, the statement with the highest average score was the lecturer's interesting way of presenting the lecture (M = 6.23), followed by satisfaction and general interestingness of the lecture (M = 6.00), and the effectiveness of the learning media (M = 6.05). In contrast, two aspects obtained low scores, namely the lecturer's friendliness (M = 3.45) and the interestingness of the material (M = 3.36), which indicates areas for improvement in the implementation of PjBL.

Student Learning Outcome Score Data Results

Learning outcome data consists of assignment grades, midterm exams, and final exams. This data is calculated into a final score and categorized into three categories: High, Medium, and Low. Based on Table 2 shows that 18 out of 22 students or 81.8% are included in the Medium and High categories, which means that academic achievement is relatively good after the implementation of PjBL in learning Analytical Geometry. It is also seen that students included in the High category have consistent scores between assignments, mid-term exams, and final exams, while students included in the Medium and Low categories show fluctuations in scores, especially in the written exam component.

Table 2. Distribution of Student Learning Outcomes Categories

Category	Number of Students	Percentage
High (≥ 80)	12	54.5%
Medium (65–79)	6	27.3%
Low (< 65)	4	18.2%
Total	22	100%

Key Themes from the Open-Ended Questionnaire

The open-ended questionnaire results were analyzed using thematic analysis techniques, which involved axial coding and selective coding. Based on the analyzed data, several main themes emerged, as presented in Table 3.

Table 3. Summary of Open Questionnaire Results Themes

Main Theme	Description of Findings
Project-based learning	Students feel more active, engaged, and motivated to explore concepts through project assignments.
Utilization of technology	The use of GeoGebra, pen tablets, and digital media helps visualize and understand concepts.
Students developed more flexible problem-solving strategies	Students gain a quick and efficient way to solve problems without losing conceptual understanding.
Development of critical and creative thinking	Learning trains students to think flexibly and analytically, and not be fixated on a single procedure.
Motivation and learning atmosphere	Collaborative discussion elements and lecturer communication create a fun learning atmosphere.
Areas for improvement	Boredom due to the large number of Learning Activity Sheets, lack of hands-on practice, and unsystematic explanations.

Based on the data in Table 3, it can be seen that students received the learning process well. Although there were criticisms, particularly regarding learning management and assignment load, they consistently stated that PjBL provided an innovative and enjoyable learning experience.

Initial Integration of Quantitative and Qualitative Findings

This integration was a deliberate process to strengthen the results of the mixed methods research. Initial integration was carried out by comparing qualitative and quantitative research data. All results from closed-ended questionnaires, open-ended questionnaires, and learning outcome scores were compared to obtain a stronger picture of the research findings.

Table 4. Integration of Quantitative and Qualitative Findings

Aspects	Closed-End Questionnaire	Open-End Questionnaire	Learning Outcomes
Lecturer Presentation	High score (6.23)	Deemed innovative and engaging	Improved conceptual understanding
Learning Media	High score (6.05)	GeoGebra and Learning Activity Sheets are very helpful	Supports academic achievement
Learning Satisfaction	High score (6.00)	Students feel happy and motivated	Majority of scores are moderate–high
Lecturer Attitude	Low score (3.45)	Complained about lack of systematicity	showed limited contribution to students' perceptions
Material	Low score (3.36)	Material is interesting, packaging needs improvement	Learning outcomes remain relatively good

This integration (as shown in Table 4) consistently demonstrates that positive perceptions of the Project-Based Learning (PjBL) method and media align with learning outcomes, while interpersonal aspects and material presentation are weaknesses that need improvement. To gain a comprehensive understanding of the impact of Project-Based Learning (PjBL) on students' problem-solving skills, quantitative and qualitative findings were integrated using a joint display approach. This integration aims to demonstrate the relationships, consistencies, and differences in meaning between the two types of data as a basis for further interpretation.

Interpreting the Impact of Project-Based Learning on Students' Problem-Solving Approaches

The findings of this study indicate that the implementation of Project-Based Learning (PjBL) in an undergraduate Analytical Geometry course is associated with positive student perceptions and generally moderate to high levels of academic achievement. However, rather than interpreting these outcomes as direct evidence of improved problem-solving ability, the mixed-methods analysis provides insight into how PjBL shapes students' approaches to solving analytical geometry problems.

The integration of quantitative and qualitative findings suggests that PjBL supports problem-solving primarily by increasing students' cognitive engagement and encouraging exploration beyond routine procedures. This interpretation aligns with Schoenfeld's (2016) view that mathematical problem-solving involves monitoring, decision making, and strategic control, rather than the mechanical application of algorithms. In this study, project tasks required students to analyze open-ended problems, make representational choices, and revise solutions iteratively, thereby fostering reflective engagement with mathematical ideas. Previous meta-analytic and design-based studies have also reported positive effects of project-based learning on students' mathematical problem-solving abilities (Capraro et al., 2013; Yunita et al., 2022).

Emergence of Efficient Problem-Solving Strategies

One of the most salient qualitative findings is the emergence of what students described as more efficient solution strategies in solving analytical geometry problems. From a theoretical standpoint, these strategies should not be interpreted as superficial or shortcut reasoning in a negative sense. Instead, they reflect a movement toward more efficient and context-sensitive problem-solving approaches.

Lithner's (2008) distinction between imitative reasoning and creative mathematically grounded reasoning provides a useful lens for interpreting this finding. The strategies reported by students in this study were typically grounded in conceptual understanding and supported by visualization, rather than rote imitation of worked examples. This suggests that PjBL may facilitate a transition from imitative reasoning toward more flexible strategic reasoning, although this transition was inferred from qualitative evidence rather than measured directly.

This interpretation is consistent with Star and Rittle-Johnson's (2009) assertion that strategy flexibility is a key indicator of deep mathematical understanding. By engaging in project work that allowed multiple solution paths, students were encouraged to compare approaches, evaluate efficiency, and select strategies that were most appropriate to the problem context.

The emergence of efficient problem-solving strategies observed in this study is consistent with previous research on activity-based and worksheet-supported learning designs. Earlier studies have demonstrated that structured learning tasks can facilitate students' ability to connect representations, articulate reasoning, and adopt flexible approaches to solving mathematical problems (Kosasih et al., 2023; Ulpah et al., 2023). In the present study, similar patterns were evident as students reported developing efficient solution strategies referred to as more efficient solution strategies that were grounded in conceptual understanding rather than rote procedures. These findings align with reports from UJMES that highlight the role of project-based activities in fostering students' mathematical problem-solving abilities through meaningful engagement and contextualized tasks (Prafita et al., 2025). Together, these results suggest that Project-Based Learning can support a shift toward more strategic and reflective problem-solving approaches when accompanied by appropriate instructional scaffolding.

Role of Visualization and Technology in Analytical Geometry

The use of dynamic visualization tools, particularly GeoGebra, played a central role in students' learning experiences. Both quantitative and qualitative findings indicate that technology functioned not merely as a presentation medium, but as a cognitive tool that supported representational coordination.

From a representational perspective, Duval (2006) emphasizes that a major source of difficulty in learning geometry lies in students' inability to coordinate different semiotic registers, such as algebraic expressions and geometric figures. The findings of this study suggest that GeoGebra-assisted project work helped students overcome this difficulty by allowing them to dynamically link symbolic equations with visual representations. This finding resonates with research reported in Educational Studies in Mathematics, which highlights the role of dynamic technology in fostering conceptual understanding through simultaneous exploration of multiple representations (Sinclair et al., 2016).

Importantly, the effectiveness of technology in this study was mediated by the project-based structure of the tasks. Visualization alone did not guarantee understanding; rather, understanding emerged through cycles of exploration, discussion, and reflection embedded in the project activities.

Tensions in Implementation: Instructional Design and Facilitation

Despite the overall positive findings, the study also revealed tensions related to instructional clarity and workload management. Quantitative data indicated relatively lower student ratings for lecturer friendliness and material interest, while qualitative feedback pointed to issues such as unsystematic explanations and an excessive number of learning tasks.

These findings reflect challenges commonly reported in PjBL implementation. Krajcik and Blumenfeld (2006) note that PjBL places substantial demands on instructional design, classroom management, and facilitation. Without carefully structured scaffolding, project work may overwhelm students or obscure learning objectives. From this perspective, the identified challenges should be viewed not as failures of the PjBL approach itself, but as indicators of the pedagogical expertise required to implement it effectively.This tension underscores the importance of balancing open-ended exploration with clear guidance and feedback, particularly in mathematically abstract domains such as analytical geometry.

Theoretical Contributions

This study contributes to the literature on mathematics education in several ways. First, it extends existing research on Project-Based Learning by focusing on undergraduate analytical geometry, a context that has received relatively limited attention compared to school-level mathematics. Second, the study highlights the role of PjBL in supporting the emergence of efficient and flexible problem-solving strategies, thereby contributing to theoretical discussions on strategic reasoning and problem-solving processes.

Third, by employing a mixed-methods approach with joint displays, the study demonstrates how quantitative trends and qualitative insights can be integrated to explain learning processes rather than merely describe outcomes. This methodological contribution aligns with calls for more integrative research designs in mathematics education that move beyond isolated measures of achievement.

Limitations and Implications for Future Research

While the findings provide valuable insights, several limitations should be acknowledged. The study involved a relatively small number of participants from a single institution, which limits the generalizability of the results. Additionally, problem-solving approaches were inferred primarily from qualitative self-reports rather than direct observation or standardized problem-solving instruments.

Future research could build on this study by employing experimental or quasi-experimental designs, incorporating validated measures of mathematical problem-solving, and examining how different forms of project design influence students' strategic reasoning. Longitudinal studies may also provide insight into whether the observed shifts in problem-solving approaches are sustained over time.

Table 5. Joint Display of Project-Based Learning Impact Mechanisms

PjBL Components	Student Activities	Quantitative Evidence	Qualitative Evidence	Mechanism
Contextual project	Open problem analysis	Dominant learning outcomes medium–high	Problems are broken down in stages	Understanding the problem
GeoGebra Visualization	Graph exploration	Media mean = 6.05	"I can see a solution"	Strong representation
Compare solutions	Compare solutions	High satisfaction	Alternative strategies emerge	Strategic flexibility
Lecturer Feedback	Revise solutions	Lecturer presentation mean = 6.23	Learning from mistakes	Monitoring & evaluation

The joint display in Table 5 shows that the impact of PjBL on problem-solving skills is not direct, but rather mediated by students' exploratory activities, visualization, and strategic reflection. The findings of this study have several practical implications for mathematics lecturers, particularly in the teaching of Analytical Geometry. First, project design should emphasize the interconnectedness of visual, symbolic, and contextual representations to encourage students to build an integrated understanding of concepts. Second, providing students with space to explore and compare various problem-solving strategies can support the development of strategic flexibility and deeper mathematical reasoning. Third, the use of structured written reflections has been shown to help students monitor their thinking processes and evaluate their solutions.

Theoretically, this study expands the understanding of how Project-Based Learning works in the context of college-level mathematics learning, specifically Analytical Geometry, by highlighting the cognitive and strategic mechanisms involved. Methodologically, the use of a mixed-methods design with joint display provides an integrative framework that allows for a coherent analysis of the relationships between quantitative and qualitative outcomes.

CONCLUSION

This study aims to explore the impact of implementing Project-Based Learning in Analytical Geometry learning on students' problem-solving skills and to understand the learning mechanisms underlying these impacts through a mixed-methods approach. This study examined the implementation of Project-Based Learning (PjBL) in an undergraduate Analytical Geometry course using a mixed-methods approach. The findings indicate that PjBL provides a learning environment that supports students' engagement with analytically demanding tasks and encourages more reflective and strategic approaches to problem-solving. Quantitative results show generally positive student perceptions of the learning methods and media, alongside moderate to high levels of academic achievement. Qualitative findings further reveal that project-based activities foster active exploration, the coordination of algebraic and visual representations, and the emergence of efficient problem-solving strategies grounded in conceptual understanding.

The integration of quantitative and qualitative findings suggests that the contribution of PjBL to problem-solving is not direct or procedural, but mediated through students' engagement in exploratory project work and the use of dynamic visualization tools such as GeoGebra. Rather than merely applying prescribed procedures, students were encouraged to analyze problems, make representational choices, and evaluate the efficiency of alternative strategies. The findings of this study also reinforce previous research indicating that learning designs centered on active tasks and structured student worksheets play a crucial role in supporting mathematical understanding and strategic problem solving. Consistent with earlier studies on worksheet development and communication-oriented mathematical tasks, the present results suggest that project-based activities encourage students to engage more deeply with representations and reasoning processes. Furthermore, evidence from UJMES confirms that Project-Based Learning contributes positively to mathematical problem-solving abilities across educational levels. Collectively, these findings position Project-Based Learning as a promising approach for supporting meaningful problem-solving processes in analytical geometry, provided that instructional design and facilitation are carefully managed. This finding reinforces the view that meaningful problem-solving in higher mathematics education involves strategic reasoning and reflective decision making. Scientifically, this study contributes by offering an integrative analytical perspective that connects AI platform typologies, pedagogical affordances, and empirical findings, thereby providing a more comprehensive foundation for future research in AI-supported mathematics learning.

From a theoretical perspective, this study contributes to the literature on mathematical problem-solving by illustrating how project-based learning can support the development of flexible problem-solving approaches in analytically complex domains. The findings align with constructivist perspectives that emphasize active knowledge construction and extend prior research by highlighting the role of representational coordination and visualization in supporting strategic reasoning. By focusing on undergraduate analytical geometry, this study adds to a relatively underexplored area of PjBL research in higher education mathematics.

Practically, the findings suggest that PjBL can serve as a viable instructional approach for Analytical Geometry courses, particularly when abstract concepts require visualization and exploration. However, the effectiveness of PjBL depends heavily on instructional design and facilitation. Lecturers should provide clear guidance, structured scaffolding, and timely feedback to support students' engagement with project tasks. The integration of digital tools such as GeoGebra should be intentionally aligned with learning objectives so that technology functions as a cognitive support rather than a mere presentation medium. Additionally, careful attention to workload and time management is necessary to prevent cognitive overload and maintain student motivation.

Several limitations of this study should be acknowledged. The relatively small sample size and the focus on a single institutional context limit the generalizability of the findings. Furthermore, students' problem-solving approaches were inferred primarily from self-reported qualitative data rather than direct observation or standardized problem-solving assessments. Future research could address these limitations by employing larger and more diverse samples, incorporating validated measures of mathematical problem-solving, and examining the long-term impact of PjBL on students' strategic reasoning. Comparative studies involving different instructional designs may also provide deeper insight into the mechanisms through which PjBL supports problem-solving in higher mathematics education.

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